

Beyond Length Matching: Electromagnetic Verification and Optimization of Differential-Pair Meanders

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Abstract—A 180-degree U-bend in a differential pair makes the inner trace shorter by πs , and the usual fix is a serpentine that adds the same length back. We test that fix with full-wave simulation instead of assuming it. We solve a controlled four-port fixture ($s = 0.508$ mm, 20 mil) in openEMS (FDTD) for 22 geometries in microstrip and stripline, 140 single-port-excited runs in all. Every result is taken through mixed-mode S-parameters to receiver waveforms. The untuned bend gives 9.27 ps of skew, against 9.45 ps from geometry and the single-line effective permittivity. Tuning that makes the two centrelines exactly equal in length removes only 44–75 % of that skew. 2.29–5.16 ps remain, and the shortfall grows with the number of excursions. Where a tuning was solved on three or more meshes its value carries a mesh-based bound (± 0.31 ps for C, ± 0.43 ps for D). The broad tuning F was solved on one mesh only. A transmission-line model built from the same lengths predicts less than 1 ps. In a serpentine experiment, folds closer together delivered less of their predicted delay (delay efficiency 0.84 with folds 1.0 mm apart, 0.68 at 0.15 mm). This is consistent with coupling between the folds. The shortfall that remains with loose folds is consistent with an effect of the corners. Neither mechanism is isolated here. The same copper in stripline, solved on one mesh, shows the same pattern. At 16 Gb/s the skew hardly closes the differential eye, from 92.9 to 91.1 % of the swing, but turns 13.6 % of the swing into common mode. Sizing the tuning from its own residual in one more EM iteration brings the skew to 0.15 and 0.56 ps for the broad and sparse tunings, but only to 1.90 ps for the dense accordion, consistent with its added amplitude coupling across its folds too. Every result is a simulation from one solver, openEMS. Nothing was measured. A coupon plan for measuring the predictions is given.

Index Terms—Differential signaling, length matching, serpentine delay line, intra-pair skew, mode conversion, mixed-mode S-parameters, FDTD, signal integrity.

I. INTRODUCTION

Length matching is the standard answer to intra-pair skew. A widely shared illustration [1] shows the inner trace of a U-bend arriving about 9.5 ps early at 20 mil pitch, corrected by a serpentine of about 1.6 mm. The geometry is right, and the delay estimate is reasonable for a common stackup. What the picture does not establish is the step from equal *length* to equal *delay*. A serpentine pulls one trace away from its partner, which changes the coupling. It also adds corners and folds that couple to each other. A CAD length report shows none of that.

This paper asks one question. When a bent pair is tuned to equal centreline length, how much of the skew is actually

removed and at what cost in reflection, mode conversion and board area? We answer it for one microstrip stackup by full-wave simulation of a controlled fixture and check it in stripline. A transmission-line model sits beside the full-wave results to separate what length explains from what it does not (Fig. 8). That a meander delays less than its length is known for single-ended delay lines (Section III). What we add is a controlled quantification of the skew that length matching leaves when it is used to correct a differential bend, its dependence on the tuning topology, and a test of sizing the correction from one field solution.

Contributions: (i) a fixture and parameter set in which only the tuning changes (Section IV); (ii) four-port full-wave extraction with mesh convergence and an uncertainty in picoseconds (Sections V, VII); (iii) the simulated delay efficiency of four tuning styles and two serpentes, in both media (Section VIII); (iv) an EM-informed sizing step and a Pareto comparison (Section IX); (v) a coupon plan to check the predictions in hardware (Section X). The scripts and data are to be released with the report (Section XII).

What is and is not claimed. Every curve and number in Sections VIII and IX is a simulation result. Section IV is analytical and is labelled so. No hardware was built and nothing was measured. Mode conversion is reported as S_{cd21} and as common-mode voltage at the receiver. Neither is radiated emission, and the paper makes no emission claim.

II. BACKGROUND

A symmetric coupled pair carries two quasi-TEM modes. The odd mode carries the differential signal and the even mode the common mode. In microstrip the odd mode has more of its field in air, so its effective permittivity is lower and it travels faster [2]. A length mismatch between the two traces converts part of a differential wave into common mode, which then travels at the even-mode velocity.

Mixed-mode S-parameters [3], [4] express this directly. S_{dd21} is differential transmission, S_{dd11} differential reflection and S_{cd21} the common mode that leaves port 2 when differential mode enters port 1. For a pair whose two paths differ only by a delay $\Delta\tau$ and whose modes do not interact,

$$\left| \frac{S_{cd21}}{S_{dd21}} \right| = \left| \tan \frac{\omega \Delta\tau}{2} \right|, \quad (1)$$

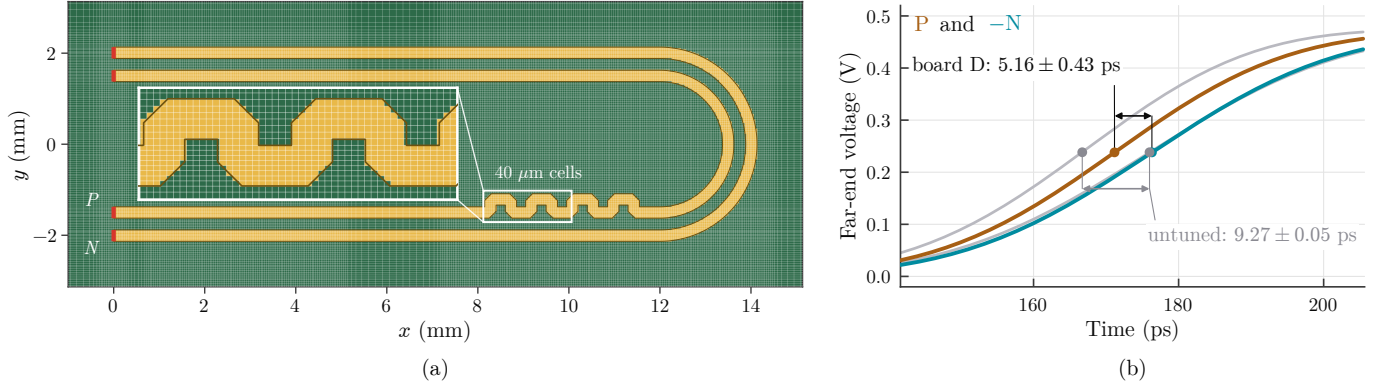


Fig. 1. Board D, one of the study’s fixtures. (a) Seen from above as openEMS meshes it, cut to the board outline: the substrate (green), the copper (gold) and the four 50Ω lumped ports (red). The medium mesh the runs used is drawn over it, graded from 40 μm cells over the copper to 150 μm away from it. Cells whose centre lies in the copper are filled. The inset enlarges two of the four excursions so that the staircase shows. The excursions make P exactly as long as N. (b) What openEMS finds at the receiver for a 50 ps edge: P still arrives 5.16 ± 0.43 ps before N, against 9.27 ps for the untuned bend (grey).

which we invert to give a frequency-resolved skew. Earlier work on single-ended serpentes found that coupling between adjacent segments makes a meander electrically shorter than its length [5], [6]. We test whether that matters at the scale of a single bend correction, and in a pair.

III. RELATED WORK

Serpentine delay lines. Wu and Chao showed that coupling between the parallel sections of a single-ended serpentine lets part of a signal arrive ahead of the rest. They called it the laddering wave, and it makes the line delay less than its length implies [5]. They also proposed a flat spiral layout that spreads the coupled noise in time [7]. Rubin and Singh modelled and measured the same reduction of delay in circuit-board meanders [6]. Guo et al. compared single-ended and differential serpentine and flat spiral delay lines by their TDR and TDT waveforms and eye diagrams. The differential flat spiral performed best [8]. Shiue et al. reduced the coupling between sections with guard traces grounded at both ends [9]. These works concern delay lines, in which the meander is the purpose of the route.

Bent differential pairs. The mode conversion of a bend is mostly addressed by compensating structures at the bend. They include shunt compensation capacitance with an extracted equivalent circuit [10], compensation inductance [11], mushroom structures on the inner line [12] and a rat-race-based method that balances any unbalanced four-port [13]. Lim et al. instead narrowed the outer line so that the delays of the two lines match, and compared that with compensation by a serpentine inner line [14]. These works report the bandwidth over which conversion is suppressed, mostly against an uncompensated bend. They validate their structures by simulation and measurement. To our knowledge they do not report how much of a bend’s skew a length-matched serpentine leaves, nor how that residual depends on the serpentine’s topology.

EM-based design. Space mapping and related surrogate methods drive a geometry from full-wave solutions with few expensive evaluations [15]. The sizing step of Section IX is a

far simpler use of one field solution to correct a design rule, and we claim nothing beyond it.

Our contribution is therefore narrow: a controlled fixture in which only the tuning changes, the residual skew and mode conversion of length-matched corrections of one bend in full-wave simulation with numerical bounds, a line model that separates what length explains and the limits of a one-step correction.

IV. GEOMETRY AND ANALYTICAL ESTIMATES

The bend. Two concentric arcs that turn through θ with centreline spacing s differ in length by

$$\Delta L = (R + s)\theta - R\theta = s\theta, \quad (2)$$

independent of the radius R (Fig. 2). For a single 180-degree U-turn and $s = 20$ mil = 0.508 mm, $\Delta L = \pi s = 1.596$ mm = 62.8 mil. Two 180-degree turns in the same direction double this ($2\pi s$). Two in opposite directions cancel in length, but between them the pair carries the skew and its common mode. The nominal delay difference is

$$\Delta t = \Delta L \sqrt{\epsilon_{\text{eff}}} / c. \quad (3)$$

With $\epsilon_{\text{eff}} = 3.8$, Eq. (3) gives 10.4 ps, and the illustrated 9.5 ps corresponds to $\epsilon_{\text{eff}} = 3.18$, a faster line. For our stackup the closed form of Hammerstad and Jensen [16] gives a single (uncoupled) line $\epsilon_{\text{eff}} = 3.154$ and $Z_0 = 64.2 \Omega$, so $\Delta t = 9.45$ ps. The odd- and even-mode values we extract below ($\epsilon_{\text{eff}} = 2.97$ and 3.50) bracket this at 9.17 and 9.95 ps. Eq. (3) uses one velocity, and a coupled pair has two. That is the first thing it omits.

The tuning rule. N rectangular excursions of amplitude A (Fig. 3) add

$$\Delta L_{\text{add}} \approx 2NA \quad (4)$$

of centreline. Eq. (4) omits five things. The first is geometry, the length each corner treatment removes (a 45-degree chamfer of leg c removes $(2 - \sqrt{2})c$ per corner, $4(2 - \sqrt{2})c$ per excursion), and we correct for it. The other four are electrical:

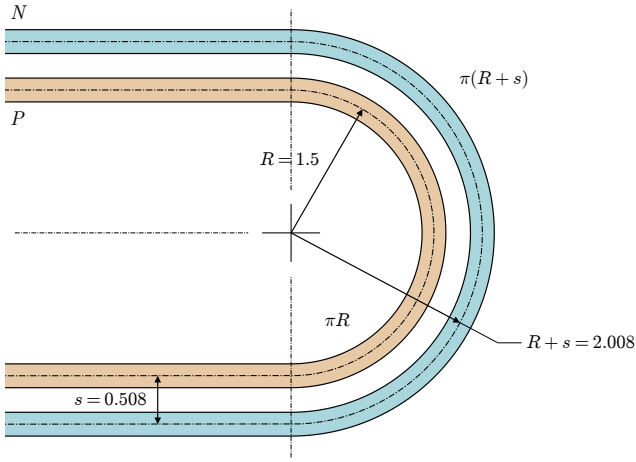


Fig. 2. The 180-degree bend of fixture B, dimensions in mm. Centrelines are dash-dotted, and P is the inner trace. The arc lengths πR and $\pi(R+s)$ differ by πs whatever the radius.

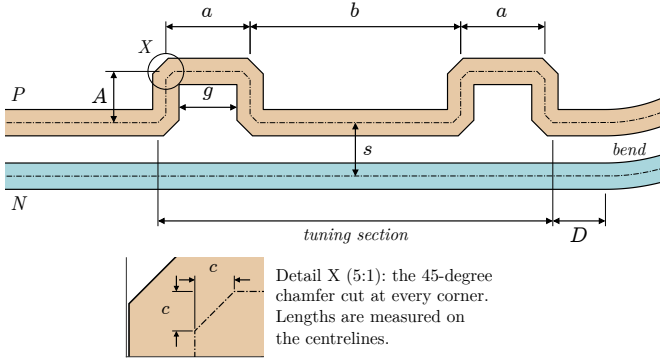


Fig. 3. One tuning section (C, $N = 2$) and the symbols used for it: amplitude A between centrelines, top a and gap b along them, fold edge spacing g , pitch s , distance to the bend D . Every corner carries a 45-degree chamfer of leg c (detail X, 5:1).

the change of mode where the tuned trace leaves its partner, the change in differential impedance over that stretch, the corner as a discontinuity whose electrical length differs from its centreline, and coupling between the folds of the excursion. In our fixtures the amplitude is solved so that the two centrelines are equal to well under a micrometre after the chamfers are cut. The 2NA rule alone would leave the dense accordion 0.70 mm short.

The fixture. One microstrip layer over a return plane (Fig. 5, Table I): $h = 0.2$ mm, $\epsilon_r = 4.3$, $w = 0.25$ mm, $s = 0.508$ mm. Every bent variant uses the same U-bend ($R = 1.5$ mm for P), the same 12 mm legs and the same launch plane, so only the tuning differs (Fig. 4): A, a straight pair of the U-bend's mean length; B, the untuned bend; C, two sparse excursions next to the bend; C far, the same two excursions next to the launch; D, a dense accordion of four; F, one broad trombone; and E, a pair-level raised-cosine excursion of both traces together, a control whose net turning angle is zero and which therefore leaves the length difference unchanged. Each single-trace tuning is cut into P on the leg leading into the bend and

TABLE I
PARAMETERS. ALL VALUES ARE **PROPOSED** FOR THIS STUDY. NONE IS CALIBRATED TO A LAMINATE OR A FABRICATOR.

Parameter	Unit	Value	Range or note
Dielectric height h	mm	0.2	fixed
Relative permittivity ϵ_r	–	4.3	fixed
Loss tangent at 5 GHz	–	0.02	fixed, $\propto 1/f$
Copper thickness / σ	mm / MS/m	0.035 / 58	sheet model
Trace width w	mm	0.25	fixed
Pair pitch s (centre to centre)	mm	0.508	fixed, 20 mil
Bend radius of P, R	mm	1.5	0.75, 1.5, 3.0
Leg length (launch to bend)	mm	12.0	fixed
Excursions N	–	1, 2, 4	F, C, D
Amplitude A	mm	solved	0.29–1.08
Excursion top / gap	mm	0.45–3.0	per family
Corner chamfer	mm	0.075	fixed
Stripline plane spacing	mm	0.7	trace midway
Distance to bend D	mm	0.5	0.5 or at launch
Serpentine added length	mm	6.0	fold gap 1.0, 0.15
Edge (10–90 %)	ps	35–300	post-processing
Bandwidth	GHz	0–30	Gaussian pulse
Finest cell (coarse/medium/fine)	μm	60/40/27	convergence

turns away from N. Every route length change is intentional and listed in Table II. Clearance between P and N copper is checked by our generator and by KiCad 10's DRC (kicad-cli pcb drc, 0.15 mm rule), and no variant has a clearance or short violation. KiCad does not check spacing within one net, so the fold spacing is set by the parameters, 0.20 mm edge to edge in D.

V. NUMERICAL MODEL

A. Stage II: a two-conductor line model

For the pair, the coupled telegrapher equations are

$$\frac{d\mathbf{V}}{dx} = -(\mathbf{R} + j\omega\mathbf{L})\mathbf{I}, \quad \frac{d\mathbf{I}}{dx} = -(\mathbf{G} + j\omega\mathbf{C})\mathbf{V}, \quad (5)$$

with 2×2 per-unit-length matrices whose off-diagonal terms are the coupling [2]. For a symmetric pair the even and odd modes diagonalise them. We calibrate the modes on the full-wave straight fixture A rather than assume a cross-section. Its S_{dd} (100 Ω) and S_{cc} (25 Ω) are each the two-port of a uniform line, and the extraction of Eisenstadt and Eo [17] gives $Z_{\text{odd}} = 53.4 \Omega$ ($Z_{\text{diff}} = 106.7 \Omega$), $Z_{\text{even}} = 67.7 \Omega$ ($Z_{\text{comm}} = 33.8 \Omega$), $\epsilon_{\text{eff}} = 2.97$ (odd) and 3.50 (even), with loss and dispersion over the whole band (the values quoted are means from 0.5 to 10 GHz). Each variant is then a cascade of chain matrices, coupled where the traces run together and uncoupled single lines (Hammerstad–Jensen parameters) where P leaves N or where N's excess lies in the bend, spread over 18 slices. The model carries every length and none of the rest. Corners, coupling between a meander's folds or across the U, and the bend as a discontinuity are all missing from it, so where it disagrees with full wave the cause is something other than length.

B. Stage III: full-wave extraction

The solver is openEMS [18], [19], an equivalent-circuit FDTD code, version v0.37.0-rc3-31-g6761a36. Its Python bindings were not installed on the host, so the model is written directly in the XML vocabulary those bindings produce, which we had checked against this openEMS version before. The

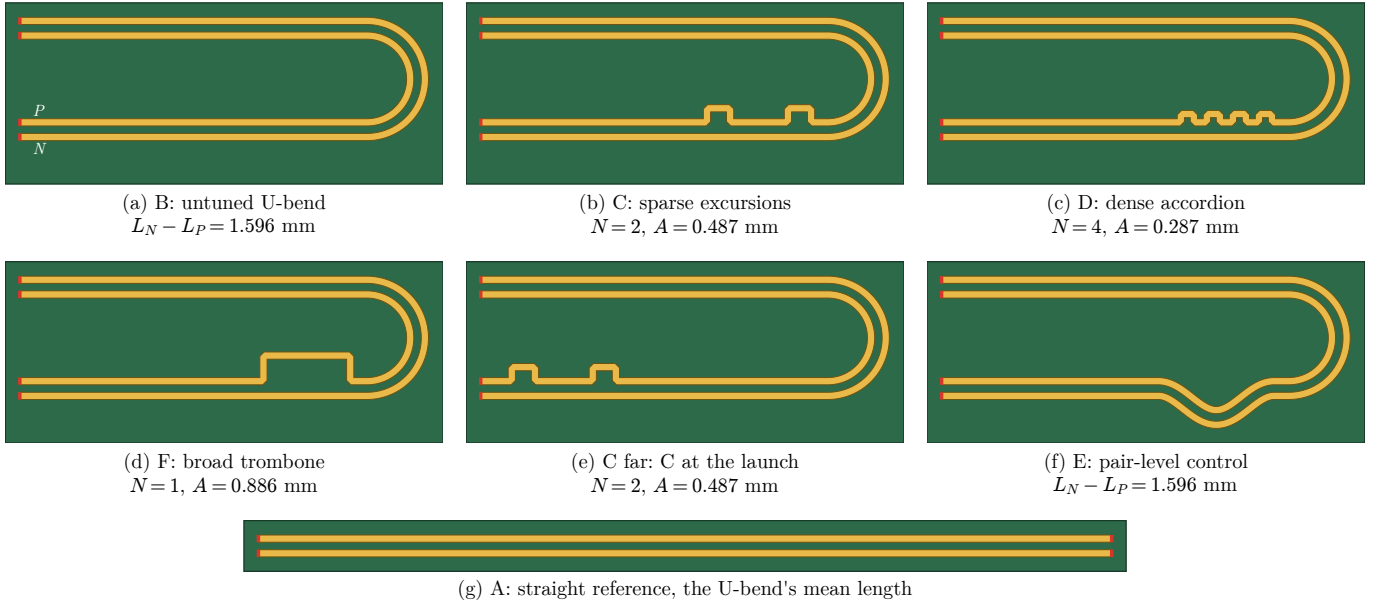


Fig. 4. The variants as openEMS is given them, all at one scale: the copper polygons (gold) and the four $50\ \Omega$ lumped ports (red) on the substrate. In every tuned variant (b–e) P and N are exactly equal in centreline length; N is the number of excursions and A the solved amplitude. A (g) is straight, as long as the U-bend’s mean centreline.

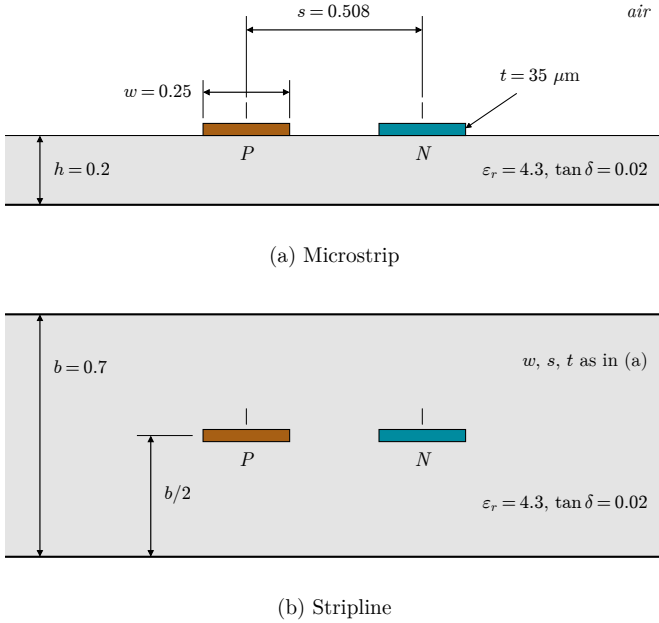


Fig. 5. Cross-sections at true scale, dimensions in mm. The stripline variants are the same copper midway between two planes.

signal copper is a conducting sheet ($35\ \mu\text{m}$, $58\ \text{MS/m}$) drawn from the generator’s polygons. The return plane is the PEC floor of the domain and the dielectric is a lossy box (Table I). openEMS models the loss as a fixed conductivity, so $\tan\delta$ falls as $1/f$. The other five faces are first-order Mur absorbing boundaries [20], at least $2\ \text{mm}$ ($10h$) from any copper laterally and $2.5\ \text{mm}$ above it (Fig. 7).

Each trace end has a $50\ \Omega$ lumped port from the plane to the copper, on the launch plane $x = 0$ (Fig. 6). Ports are numbered

in scikit-rf’s mixed-mode order [21]: 1 = P near, 2 = N near, 3 = P far, 4 = N far. Each variant takes four runs, each exciting one port with the other three passive [22], with a Gaussian pulse covering 0 to 30 GHz. From each port’s voltage u_k and current i_k (into the structure), the power waves [23] $a_k = (u_k + Z_0 i_k)/(2\sqrt{Z_0})$ and $b_k = (u_k - Z_0 i_k)/(2\sqrt{Z_0})$ give $S_{kj} = b_k/a_j$. The result is written as a Touchstone .s4p from DC to 30 GHz in 10 MHz steps.

The mesh is rectilinear and graded. Its finest cells lie over the copper regions: 60, 40 and $27\ \mu\text{m}$ for the coarse, medium and fine levels, with every straight trace edge on a grid line. Away from the copper the cells grow by at most 1.3 per cell, to 0.15 mm laterally and 0.3 mm in the air. These limits are the same at every level, so only the copper regions are refined. All bent variants share one mesh per level, built over the union of their copper, so discretisation error in the copper they share is common to them. The medium mesh of the bent fixtures has 1.24 M cells and the fine one 2.96 M. The arcs and chamfers are staircased (Fig. 6b).

The pulse’s DC content leaves a static field in the domain whose energy settles near $-38\ \text{dB}$, so no run reaches an energy end criterion. A pulse without DC content does reach one, but it measures nothing useful below about 2 GHz. Every run therefore stops at a fixed 2 ns of simulated time, by which the port voltages have fallen at least 98 dB below their peak (Table III). On fixture B (coarse mesh, port 1 excited) a 7 ns record changes the low-frequency S-parameters by about 10^{-3} .

VI. ANALYSIS METRICS

Mixed mode. `se2gmm(p=2)` on the order above gives $d_{\text{near}}, d_{\text{far}}, c_{\text{near}}, c_{\text{far}}$, so $S_{dd11} = S_{00}$, $S_{dd21} = S_{10}$, $S_{cd21} = S_{30}$ (zero-based), with reference impedances of $100\ \Omega$ differential

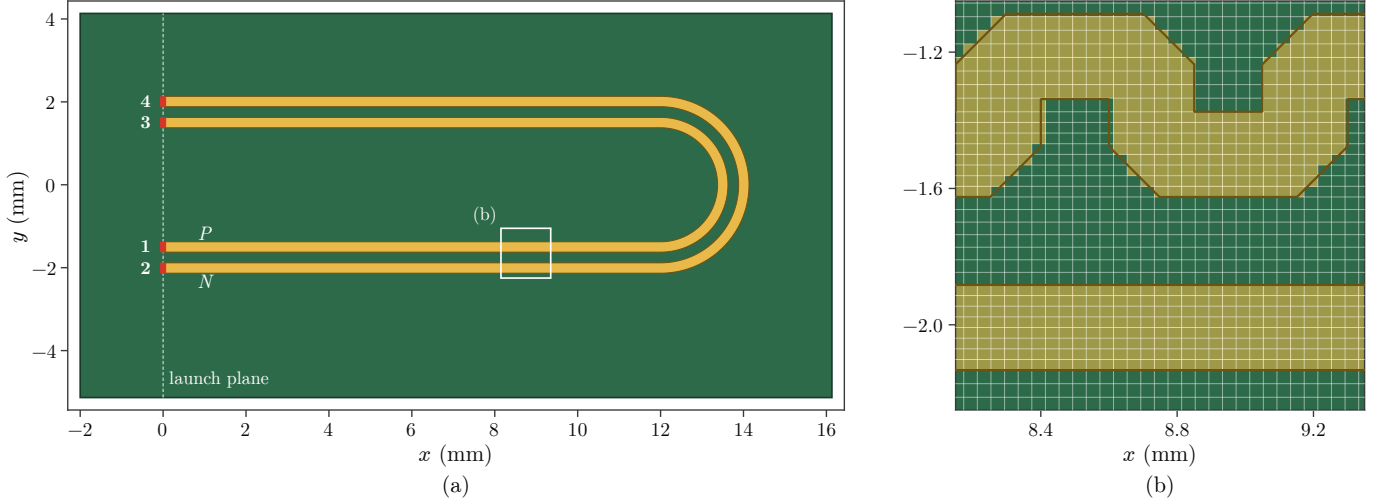


Fig. 6. (a) Fixture B in its computational domain: the substrate extends 2 mm past the copper on every side, and the ports, numbered in mixed-mode order, sit on the launch plane. (b) The medium mesh ($40\ \mu\text{m}$ finest cell) over D's first excursions. Cells whose centre lies in the copper are filled, so the 45-degree chamfers are staircased.

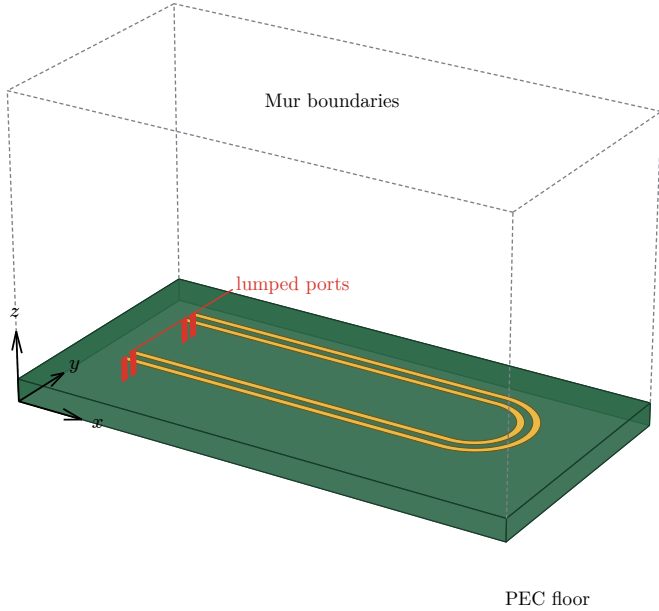


Fig. 7. The model of fixture B in three dimensions, height drawn 4 times its true scale: the dielectric slab on the PEC floor, the copper sheets, the lumped ports from the floor to the copper, and the edges of the air box, whose five open faces are Mur boundaries.

and $25\ \Omega$ common, in scikit-rf [24]. We checked this convention against the scikit-rf 2.2.0 source rather than its rendered documentation.

Waveforms. A differential step of 1 V open circuit (+1 V and -1 V, each behind $50\ \Omega$) with a Gaussian edge of 10–90% rise time t_r drives ports 1 and 2, and ports 3 and 4 are $50\ \Omega$ to the plane ($100\ \Omega$ differential, $25\ \Omega$ common). Far-end voltages follow by superposition from the four-port in the frequency domain and an inverse FFT, with nothing fitted. $V_d = V_P - V_N$, $V_c = (V_P + V_N)/2$.

Skew is $t_{50}(N \text{ falling}) - t_{50}(P \text{ rising})$ at the far end, each at

half its own final value. Positive means P, the inner trace, arrives first. We report 50 ps edges unless stated, and 35 to 300 ps in Section VIII. Edges faster than 35 ps are not evaluated, because the simulated band ends at 30 GHz.

Delay efficiency.

$$\eta = \frac{t_{\text{skew,ref}} - t_{\text{skew}}}{\Delta L_{\text{add}} \sqrt{\epsilon_{\text{eff}}^{\text{single}}} / c}, \quad (6)$$

where the reference is the identical untuned fixture (B, or the straight serpentine reference), ΔL_{add} is the centreline the tuning adds to P, and $\epsilon_{\text{eff}}^{\text{single}} = 3.154$ (5.924 ps/mm) is the velocity a designer's length-to-delay conversion would use. $\eta = 1$ means the delay is what the length predicts. On the same basis the untuned bend has $\eta = 0.981$.

Other metrics, each over 0.1–20 GHz unless stated: worst $|S_{dd11}|$; RMS $|S_{cd21}|$; differential TDR impedance from the step response of S_{dd11} with a 35 ps edge; the frequency-resolved skew of Eq. (1); a PRBS7 eye at 10 Gb/s (50 ps edges) and 16 Gb/s (35 ps edges), with height at the best phase and width where the eye is open by more than 10% of the swing; and the board area of the tuning, its length times its amplitude. No interface specification is assumed, so wherever a pass or fail would need one the target is illustrative and is to be selected for the intended interface.

VII. VERIFICATION OF THE NUMERICS

Table III lists the checks for every variant. Reciprocity holds to $|S_{ij} - S_{ji}| \leq 4.5 \times 10^{-3}$ (the straight fixture, symmetric by construction, to 3.3×10^{-7}). Every four-port is passive, with a largest singular value of S over the band of 0.989 for B and no more than 0.990 for any variant. Every record ends with its port voltages at least 98 dB below their peak.

Convergence. B, C, D and the serpentine set were solved on three meshes and D on a fourth (Table IV, Fig. 10). We report Roache's grid convergence index [25] over the three finest meshes, with safety factor 1.25 and an observed order

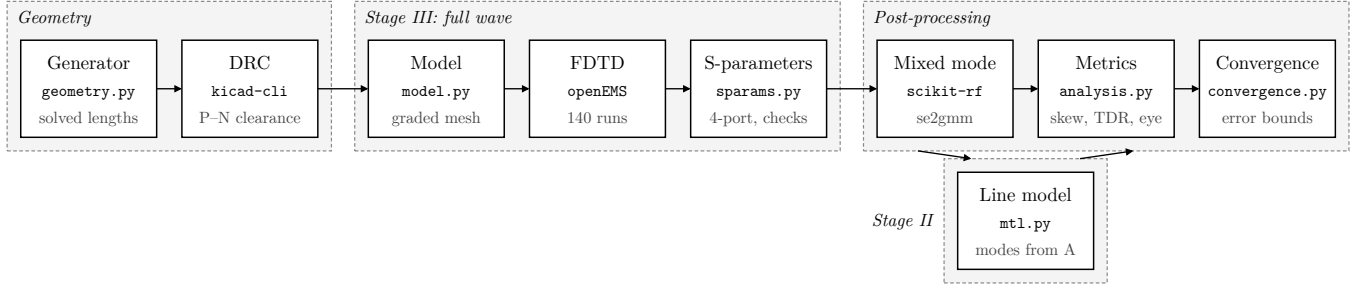


Fig. 8. The pipeline. The line model (Stage II) is calibrated on fixture A’s mixed-mode parameters and read beside the full-wave results. Each run keeps its model, its solver log and a provenance record (solver version, model hash, wall time, timesteps). Each variant keeps its .s4p and its checks.

TABLE II

METRICS, MEDIUM MESH, 50 PS EDGE. ΔL_{add} IS THE CENTRELINE THE TUNING ADDS TO P; η IS EQ. (6); MTL IS THE LINE MODEL’S SKEW; $|S_{dd11}|$ AND $|S_{cd21}|$ OVER 0.1–20 GHz; Z_{diff} FROM TDR. SKEW BOUNDS: EACH VARIANT’S OWN WHERE IT WAS SOLVED ON THREE OR MORE MESHES; † BORROWED FROM THE CONVERGED VARIANT WHOSE SMALLEST FEATURES IT SHARES, NOT A CONVERGENCE RESULT FOR THIS VARIANT (SECTION VII).

Variant	ΔL_{add} (mm)	Skew (ps)	η	MTL (ps)	V_c p-p (%)	$ S_{dd11} $ max (dB)	$ S_{cd21} $ RMS (dB)	Z_{diff} max (Ω)	Area (mm ²)
A straight	–	$0.00 \pm 0.05^{\dagger}$	–	0.00	0.00	-23.4	–	106.0	0.00
B untuned	–	9.27 ± 0.05	–	8.83	4.87	-20.7	-10.2	107.1	0.00
E pair exc.	–	$9.39 \pm 0.05^{\dagger}$	–	8.83	4.92	-21.4	-10.1	108.1	4.00
C sparse	1.596	3.59 ± 0.31	0.60	0.31	1.83	-21.6	-18.0	106.9	1.83
C far	1.596	$4.36 \pm 0.31^{\dagger}$	0.52	0.96	2.19	-19.6	-15.4	106.9	1.83
D dense	1.596	5.16 ± 0.43	0.44	0.27	2.67	-19.8	-14.9	106.9	0.95
F trombone	1.596	$2.29 \pm 0.31^{\dagger}$	0.74	0.23	1.16	-16.8	-22.0	114.4	2.79
C, EM-sized	2.213	$0.56 \pm 0.31^{\dagger}$	0.66	-2.98	0.52	-21.7	-25.2	106.7	2.40
D, EM-sized	2.484	$1.90 \pm 0.43^{\dagger}$	0.50	-4.50	0.98	-19.9	-22.6	106.7	1.31
F, EM-sized	1.991	$0.15 \pm 0.31^{\dagger}$	0.77	-1.89	0.40	-16.7	-27.8	115.0	3.41

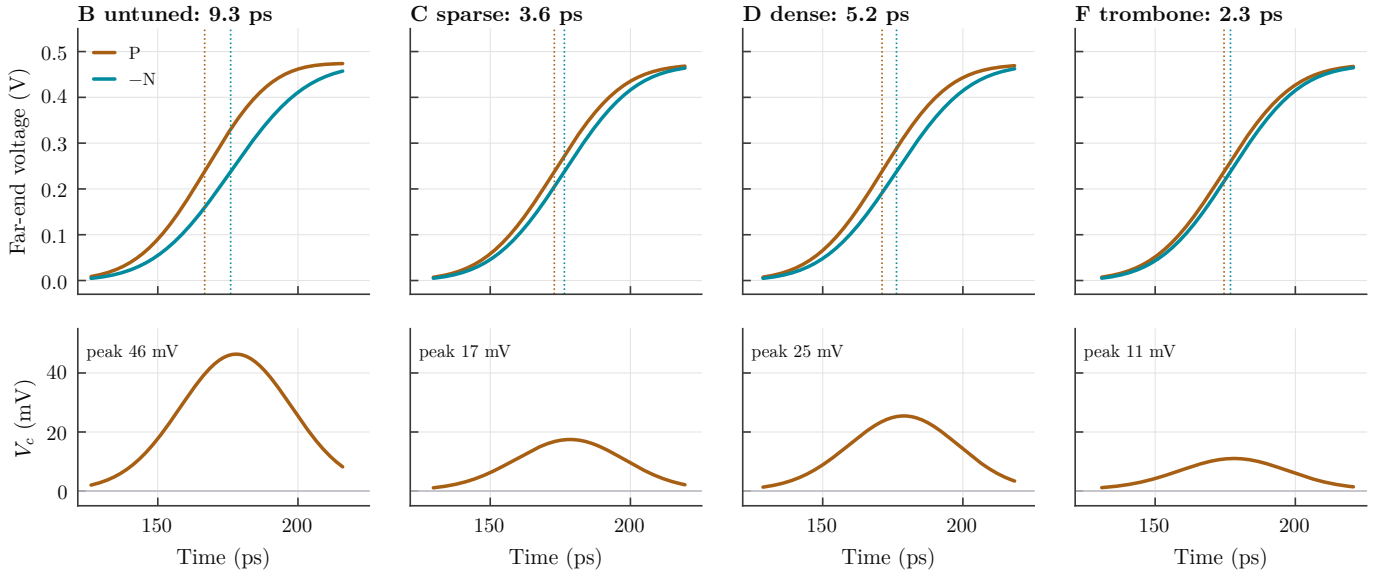


Fig. 9. Far-end P and $-N$ for a differential step (50 ps edge), and the common-mode voltage. Dotted lines mark each 50% crossing.

capped at the scheme’s formal order of 2. Where those meshes do not converge monotonically we use three times their spread. B converges monotonically, with a formal GCI of 0.02 ps on the finest mesh. D’s 75 μm chamfers span barely one cell on the coarse mesh, and its coarse value is the outlier. Over the three finest meshes it converges monotonically, to 4.82 ps. The

tight serpentine’s 0.15 mm folds still oscillate across meshes. Two caveats apply to every bound. The outer cell limits are the same at every level, so only the copper regions are refined. And where the meshes do converge monotonically the observed order is 6.0 for B and 5.3 for D, well above the scheme’s formal order of 2. The three levels are therefore not in the asymptotic

TABLE III

NUMERICAL CHECKS (MEDIUM MESH). TRUNCATION IS HOW FAR EVERY PORT VOLTAGE HAD FALLEN, RELATIVE TO ITS PEAK, OVER THE LAST 5 % OF THE RECORD.

Variant	Reciprocity $\max S_{ij} - S_{ji} $	Passivity $\max \sigma(S)$	Truncation (dB)
A straight	3.3×10^{-7}	0.989	-110
B untuned	2.4×10^{-3}	0.989	-110
C sparse	4.5×10^{-3}	0.988	-110
C far	2.8×10^{-3}	0.988	-109
D dense	3.4×10^{-3}	0.988	-110
E pair exc.	2.9×10^{-3}	0.988	-110
F trombone	2.8×10^{-3}	0.988	-110
C, EM-sized	3.7×10^{-3}	0.988	-110
D, EM-sized	3.0×10^{-3}	0.988	-110
F, EM-sized	2.0×10^{-3}	0.988	-110
Serp. ref.	6.0×10^{-5}	0.989	-110
Serp. 1.0 mm	1.8×10^{-3}	0.988	-109
Serp. 0.15 mm	2.1×10^{-3}	0.988	-109
B, R 0.75	3.5×10^{-3}	0.990	-110
B, R 3.0	2.4×10^{-3}	0.987	-108
B stripline	1.8×10^{-3}	0.989	-109
C stripline	1.4×10^{-3}	0.988	-109
D stripline	1.8×10^{-3}	0.988	-110
F stripline	2.2×10^{-3}	0.988	-109
Serp. ref. SL	2.9×10^{-5}	0.988	-99
Serp. 1.0 mm SL	1.5×10^{-3}	0.987	-98
Serp. 0.15 mm SL	1.5×10^{-3}	0.987	-98

TABLE IV

MESH CONVERGENCE OF THE SKEW (50 PS EDGE). THE FINEST-MESH GCI IS THE FORMAL VALUE. THE 40 μm BOUND IS WHAT THE COMPARISON TABLES CARRY (SECTION VII).

Variant	60 μm (ps)	40 μm (ps)	27 μm (ps)	18 μm (ps)	GCI finest	40 μm bound	Convergence
B untuned	9.54	9.27	9.25	–	± 0.02	± 0.05	monotonic
C sparse	3.69	3.59	3.61	–	± 0.31	± 0.31	oscillatory
D dense	4.45	5.16	4.88	4.84	± 0.03	± 0.43	monotonic
Serp. ref.	0.03	0.03	0.03	–	± 0.00	± 0.00	monotonic
Serp. 1.0 mm	-32.41	-29.99	-29.69	–	± 0.31	± 0.68	monotonic
Serp. 0.15 mm	-23.63	-24.12	-22.98	–	± 3.40	± 3.40	oscillatory

range. That is why the order is capped, and why the finest-mesh GCIs are formal values rather than demonstrated ones. One cause is that only the straight legs have their copper edges on mesh lines. The tops and verticals of the tuning sections do not, so each level sees an amplitude and a vertical's width quantized to its own cell, by up to half a cell. The comparison tables quote the medium mesh throughout, so each medium value carries its own bound, 1.25 times its distance from the extrapolated value, or three times the spread where the meshes oscillate (Table IV). That gives ± 0.05 ps for B, ± 0.31 ps for C and ± 0.43 ps for D, whose medium value runs high by about that much. A variant solved on the medium mesh only carries the bound of the converged variant whose smallest features it shares: C's for C far, F and the EM-sized C and F; D's for the EM-sized D; B's for E and the other bend radii. These borrowed bounds are not convergence results for those variants. Table II marks them with $\dagger$, and Fig. 11 draws them dashed. Because a wide bound can hide a trend or invent one, we also judge every conclusion mesh by mesh. Table V gives η on each mesh, against the untuned reference on the same mesh.

TABLE V

DELAY EFFICIENCY η (Eq. 6) ON EACH MESH, AGAINST THE UNTUNED REFERENCE ON THE SAME MESH. D AT 18 μm HAS NO B AT THAT LEVEL AND IS COMPARED WITH B AT 27 μm .

η on each mesh	60 μm	40 μm	27 μm	18 μm
C sparse	0.62	0.60	0.60	–
D dense	0.54	0.44	0.46	0.47
Serp. 1.0 mm	0.91	0.84	0.84	–
Serp. 0.15 mm	0.67	0.68	0.65	–

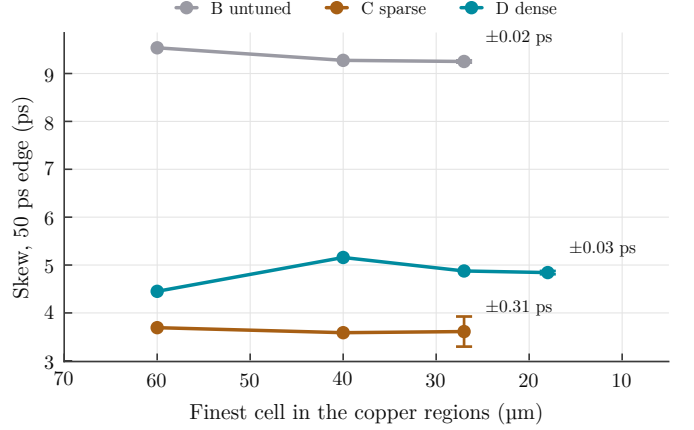


Fig. 10. Skew against the finest cell size. The bar on each variant's finest point is its formal GCI bound (Section VII).

VIII. RESULTS

All values in this section are simulated. Table II collects them.

A. The untuned bend

B shows 9.27 ps of skew at 50 ps edges, against 9.45 ps from Eq. (3) with the single-line ε_{eff} , so geometry predicts the bend to 1.9 % by this metric. The line model, which holds the same lengths and the two modal velocities and nothing else, reads 8.83 ps by the same metric. Part of what a 50 % crossing makes of a pure length difference in a coupled pair is the mode conversion that difference causes, so the agreement with Eq. (3) is specific to the metric. The skew does depend on the edge, 8.92 ps at 35 ps and 10.03 ps at 300 ps (Fig. 19), moving from near the odd-mode prediction (9.17 ps) toward the even-mode one (9.95 ps) as the edge slows. The bend radius matters a little where Eq. (2) says it should not, with 9.60 ps at $R = 0.75$ mm, 9.27 ps at 1.5 mm and 9.07 ps at 3 mm (Fig. 20). The pair-level excursion E leaves the skew where B has it (9.39 ps) and adds 4.3 ps of pair delay, as designed.

B. Equal length is not equal delay

Fig. 11 is the central result. Every single-trace tuning makes P and N equal in centreline length, and every one leaves skew. The broad trombone F leaves 2.29 ps and the two sparse excursions C leave 3.59 ps, both ± 0.31 ps (C's own bound, borrowed for F). The dense accordion D leaves 5.16 ps (± 0.43 ps), and 4.82 ps extrapolated over its meshes. In efficiency terms (Eq. (6)), $\eta = 0.74, 0.60$ and 0.44 for one, two

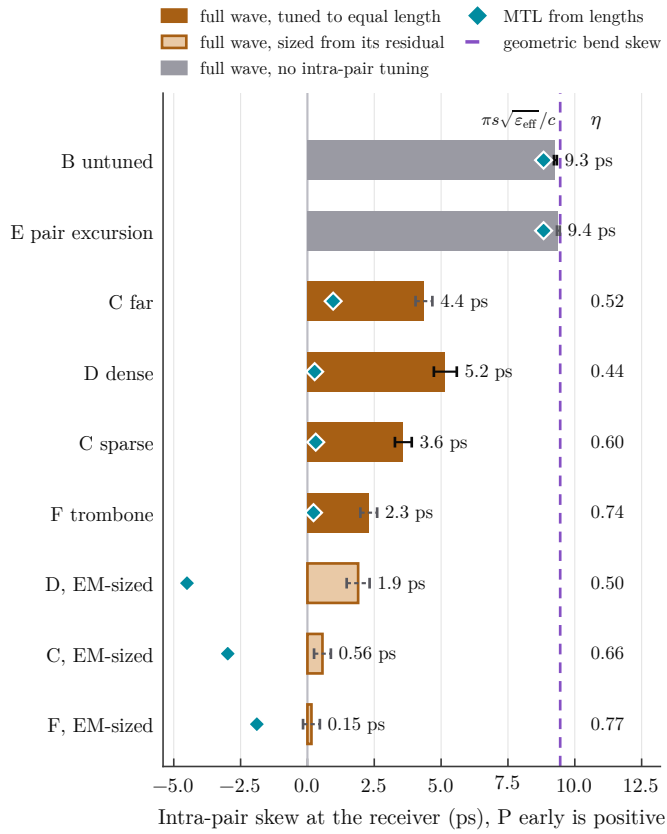


Fig. 11. Skew at the receiver, 50 ps edge. Copper: tuned to exactly equal length. Grey: no intra-pair tuning. Diamonds: the transmission-line model with the same lengths. Bars: mesh-based bounds, solid where the variant was solved on three or more meshes, dashed where the bound is borrowed from a converged variant.

and four excursions. The transmission-line model, given the same lengths, predicts 0.23, 0.31 and 0.27 ps. The waveforms (Fig. 9) show the same thing at the receiver. P still leads N, and the common-mode pulse falls from 4.87 % of the step for B to 1.16–2.67 %, not to zero.

C. Where the delay goes

The gap is consistent with two effects that the line model leaves out, coupling between the folds of a meander and its corners. The serpentine experiment varies the first. It is a straight pair with 6 mm added to P in three excursions, once with folds 1.0 mm apart (5h) and once 0.15 mm apart, the clearance rule. The two have the same corners, the same amplitude and the same added length. Their section lengths differ, 6.40 against 2.15 mm, so P also runs uncoupled from N over a different length. With loose folds $\eta = 0.84$ (0.84–0.91 across the meshes). With tight folds it is 0.68 (0.65–0.68), 16 to 24 points lower on every mesh (Fig. 13, Table V). The difference is consistent with coupling between the folds, but this pair of runs does not exclude the change in uncoupled length. The loose serpentine’s own shortfall, which the line model also misses, is consistent with an effect of its twelve corners. No run here varies the corners alone, so that attribution is by elimination. We define the shortfall per excursion of a bend

correction as the skew the line model says the tuning removed less the skew full wave says it removed, net of the 0.44 ps the model misses on the bare bend. It is 1.6, 1.4 and 1.1 ps for F, C and D. It is not a constant. It falls as the excursions get smaller and more numerous, but more slowly than their number grows, so the total shortfall still rises from one excursion to four. The three families change the excursion count, the amplitude and the fold spacing together, so this experiment does not separate them. The serpentine separates fold spacing alone, and the EM-sized variants of Section IX test the amplitude. The dense accordion D has the most corners and the closest folds, and the lowest η .

D. Stripline

The results above are all microstrip, where the odd and even modes travel at different velocities. We solved B, C, D, F and the serpentine set again in stripline to see whether that matters. The copper is the same, midway between two planes 0.7 mm apart, with the ports to the lower plane. Stripline is homogeneous, so its nominal velocity is $c/\sqrt{\epsilon_r}$ (6.917 ps/mm) and Eq. (6) uses that. Geometry predicts the bend to 0.6 % (10.97 ps against 11.04 ps). The length-matched tunings remove 81, 64 and 42 % of it for F, C and D, against 75, 61 and 44 % in microstrip, and the serpentine gives $\eta = 0.84$ and 0.64, a fold penalty of 20 points against 16 in microstrip (Fig. 15). We had expected less, since in a homogeneous medium the far-end coupling between parallel lines vanishes. One possible explanation, not tested here, is that the folds of a meander carry current in opposite directions. What couples them would then act backward along the route, and backward coupling is present in both media. These stripline runs are on the medium mesh only.

E. Placement

C far is the same tuning as C, next to the launch instead of the bend. Its arrival skew is close (4.36 against 3.59 ps), but its mode conversion is not. Between the tuning and the bend the pair carries a skew in the other direction, and the two conversions drift apart in phase with frequency. The frequency-resolved skew of C far climbs from 4.08 ps at 5 GHz to 6.49 ps at 25 GHz, while C stays near 3.55 ps (Fig. 16). Its peak $|S_{cd21}|$ to 20 GHz is -9.9 dB against -12.9 dB for C. In this one comparison, compensation next to the mismatch did better. Two placements are not a sweep of the distance.

F. Reflection, impedance and mode conversion

Where a trace leaves its partner, the differential impedance rises. The broad trombone F, whose P moves 0.886 mm further from N for 3 mm, lifts Z_{diff} to 114.4 Ω against 107.1 Ω for the untuned bend (Fig. 17), and has the worst return loss of the set (-16.8 dB against -20.7 dB, Fig. 18), so the variant with the least residual skew pays for it in reflection. The dense accordion barely moves Z_{diff} . Mode conversion follows the residual skew: across the nine bend variants, RMS $|S_{cd21}|$ falls in the same order as the skew (Fig. 12, Table II), and E’s curve lies on top of B’s.

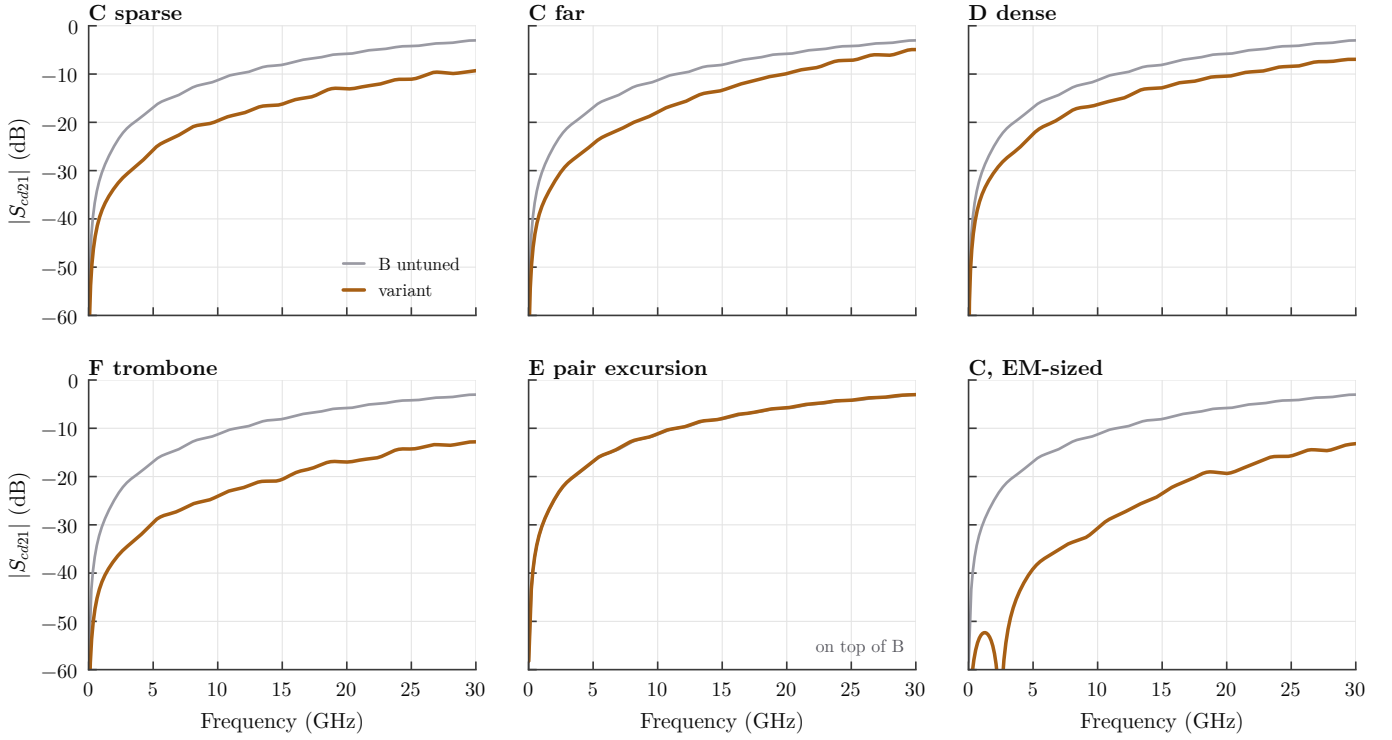


Fig. 12. $|S_{cd21}|$ of each variant (copper) against the untuned bend B (grey).

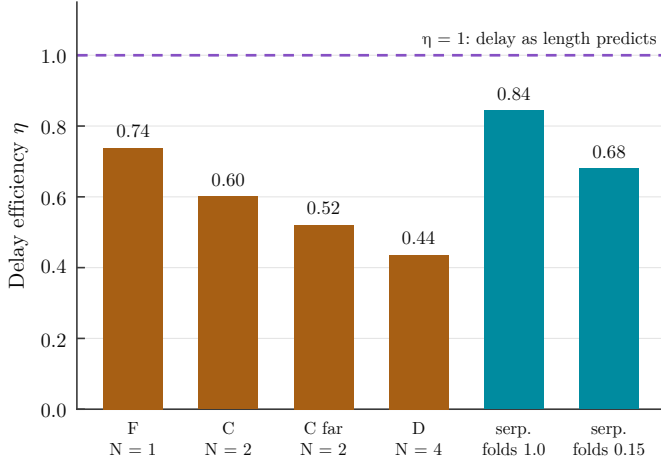


Fig. 13. Delay efficiency η (Eq. 6) of the bend corrections (copper, N excursions) and of the 6 mm serpentes (teal) with loose and tight folds.

G. What the skew does to a signal

At 16 Gb/s with 35 ps edges, the untuned bend closes the differential eye from 92.9 % to 91.1 % of the swing and leaves its width at 0.936 UI (Fig. 14). The larger effect is common mode, 13.6 % of the swing peak to peak at the receiver against 3.3 % for F. For a receiver with good common-mode rejection the skew of a single bend is a common-mode problem more than a timing one. Whether that common mode radiates depends on the structure beyond the pair, which this study does not model.

IX. EM-INFORMED SIZING

One correction step. This is not an optimizer. It takes the residual of one field solution, corrects the amplitude once and solves again. It was not iterated. If each excursion loses a fixed amount of delay to its corners and folds, more amplitude on the same excursions should recover it. We lengthened C, D and F by their own medium-mesh residual, converted to length at the bend's rate ($\Delta L_{\text{add}} = \pi s (1 + t_{\text{skew}}/t_{\text{skew},B})$). By CAD measure P is now longer than N, with 2.213, 2.484 and 1.991 mm added in total. By length alone these variants now overshoot. The line model, which holds only lengths, predicts -1.89, -2.98 and -4.50 ps. Full wave gives 0.15 ps for F and 0.56 ps for C, within the bound borrowed from C, though neither variant was solved on more than one mesh. D comes down only to 1.90 ps. It was sized from its medium-mesh residual, which Section VII puts about 0.43 ps high, so if anything it was over-corrected. This tests the fixed-shortfall model. The model holds for the trombone, roughly for the sparse tuning and not for the accordion. The extra millimetres delivered 92 % of their expected delay in F and 83 % in C, but only 62 % in D. That is consistent with its lengthened verticals coupling more strongly to one another, which no run here isolates. A broad or sparse tuning is calibrated by one field solution or one set of coupons per stackup, and a tuner could carry the per-excursion correction. A dense accordion needs iteration, which is one more reason to avoid it.

Pareto comparison. Over the bent candidates we minimise four objectives defined above, $|t_{\text{skew}}|$ (50 ps edge), RMS $|S_{cd21}|$, worst $|S_{dd11}|$ and tuning area, with no weighted score.

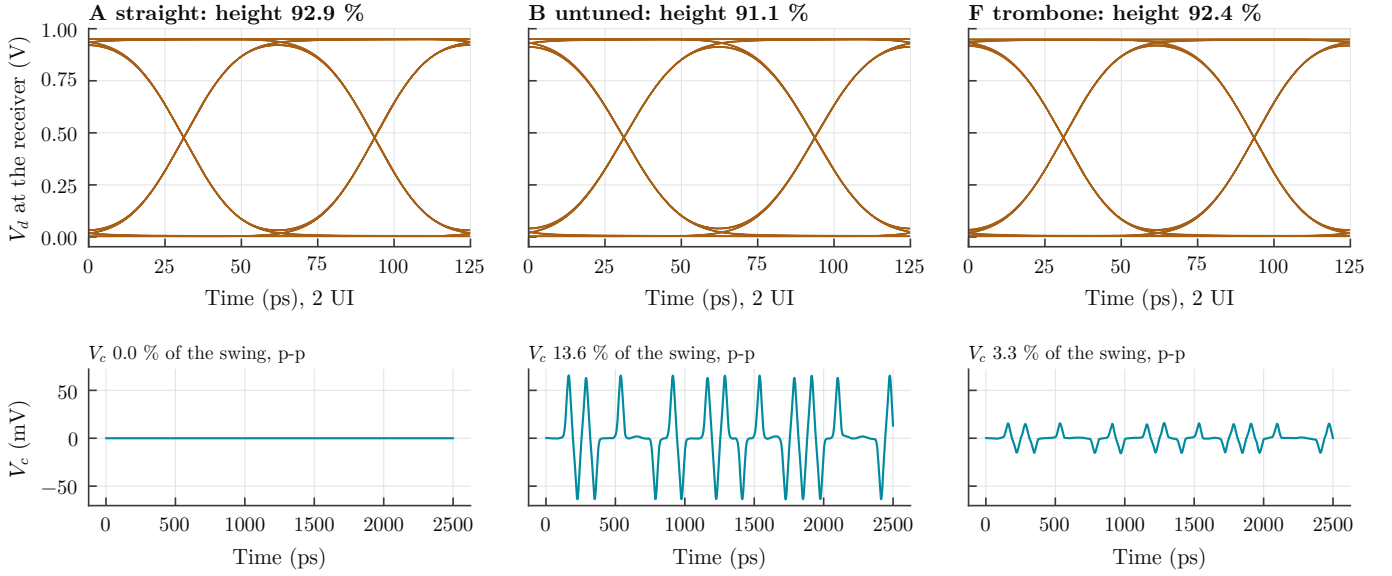


Fig. 14. 16 Gb/s PRBS7 eyes (35 ps edges) and the common-mode voltage at the receiver.

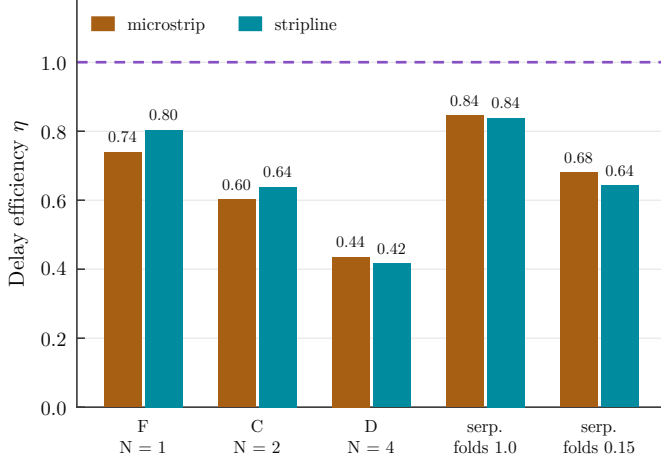


Fig. 15. Delay efficiency of the same copper in microstrip and in stripline, each against its own nominal velocity.

The non-dominated set is {B, C, D, C (EM-sized), D (EM-sized), F (EM-sized)}. B is on it only because the untuned bend has no tuning area. E adds area and removes nothing, so B dominates it. Fig. 21 shows two projections. Against area the dense accordion is cheap and weak, and against return loss the trombone trades reflection for skew.

X. PHYSICAL VALIDATION PLAN

This plan has not been executed. It calls for a coupon panel with one coupon per variant (A–F, the EM-sized variants and the serpentes), each in three copies, on one laminate and stackup. All coupons share one launch design, either edge-mount connectors or GSSG probe pads with ground vias. A 2x-thru of the launch alone on the same panel allows fixture de-embedding as in IEEE Std 370 [26]. Measure four-port S-parameters to 30 GHz on a calibrated VNA and de-embed them to the launch plane. Microsection one coupon per panel

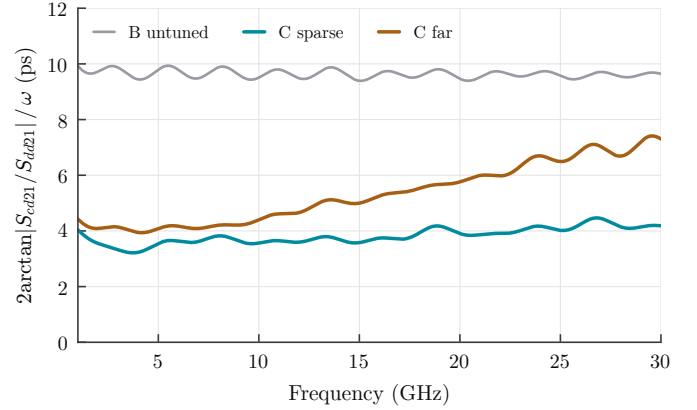


Fig. 16. Frequency-resolved skew from Eq. (1). Compensation far from the bend (C far) holds at low frequency and fails as frequency rises.

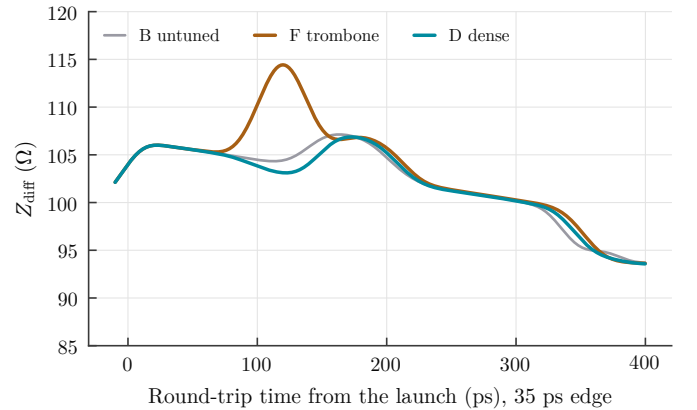


Fig. 17. Differential TDR (35 ps edge). F's uncoupled stretch shows as the peak near 120 ps.

to measure w , s , h and the copper. Extract ϵ_{eff} of the odd and even modes from coupon A and rerun the EM with

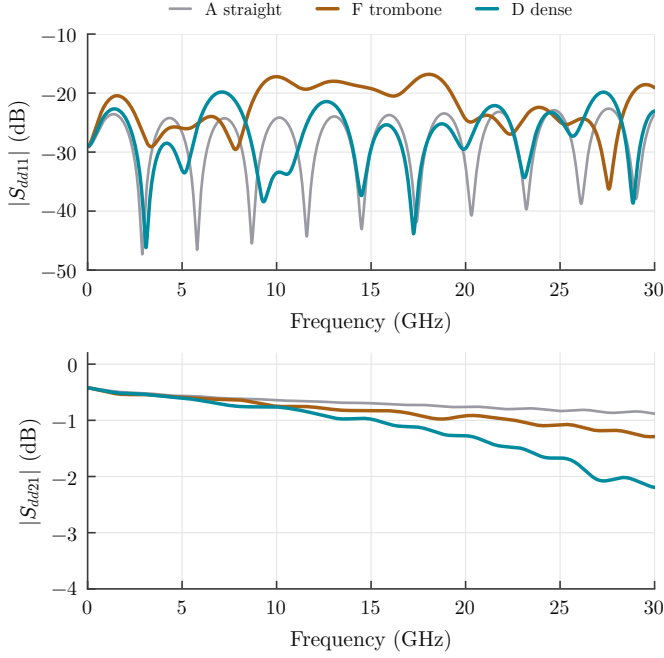


Fig. 18. Differential return and insertion loss of the straight reference A, the trombone F and the dense accordion D.

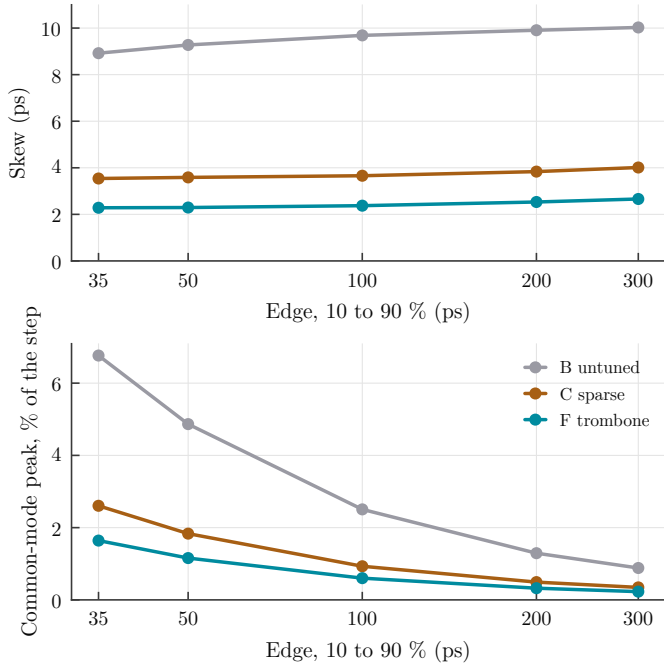


Fig. 19. Skew and peak common-mode voltage against the edge rate.

the measured dimensions before comparing. The comparison covers skew, η (Eq. 6, against B on the same panel), $|S_{dd11}|$, $|S_{cd21}|$ and TDR. Error bars combine the spread of the three copies, the VNA's stated uncertainty and the GCI here. Three predictions are to be tested. Equal-length tuning should leave 2.29 to 5.16 ps at this pitch, the shortfall should grow with the excursion count and the same should hold on a stripline panel. Near- or far-field scanning would be needed only if emission

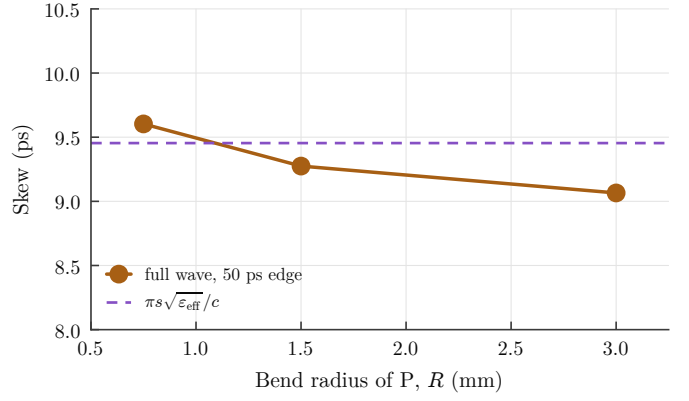


Fig. 20. Skew of the untuned bend against its radius.

is the claim, which it is not here.

XI. RECOMMENDATIONS

For edge-coupled pairs at this pitch and coupling, in microstrip and stripline and with the caveat that the numbers are simulated:

- Expect geometry to predict a bend's skew well (here within 1.9 % at a 50 ps edge, and 10.03 ps at 300 ps), and expect it to over-predict what a length-matched tuning removes. Budget a residual, or size the tuning from a field solution or a coupon.
- Prefer few excursions. The shortfall per excursion shrinks with its size, but not as fast as the count grows. A single excursion removed 75 % of the skew, four removed 44 %.
- Keep folds apart. Here folds at the clearance rule lost 16–24 points of efficiency against folds 5h apart.
- Tune next to the mismatch. The same correction at the launch matches arrival time but not phase at high frequency.
- Watch the impedance of a broad excursion. The uncoupled stretch lifts Z_{diff} , here to 114.4 Ω . Keep its amplitude small relative to the pitch or keep it short.
- A pair-level excursion does not fix intra-pair skew.
- The stripline check, on one mesh, showed the same pattern. The bend's skew matched geometry to 0.6 %, the length-matched tunings removed a similar share, and tight folds cost 20 points. The edge rate and the interface decide whether a few picoseconds matter. At 16 Gb/s here they matter more as common mode than as eye closure.

XII. REPRODUCIBILITY

The study's repository, to be published with the report, holds parameters.yaml; scripts/ with the geometry generator (and its KiCad export and DRC), the openEMS model writer, the run orchestrator with provenance, the S-parameter extraction, the line model, the analysis, the convergence study and the figure and table generators; and figures/. Every number in this report is a macro generated from build/results/results.json. We made the 140 runs on one Linux host, 27.7 h of wall time in all on single-threaded

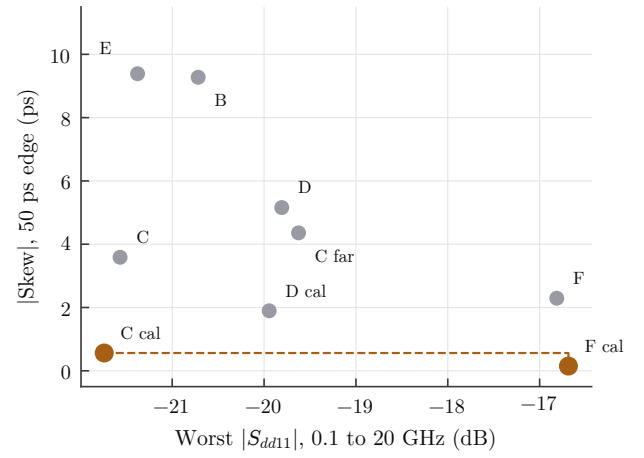
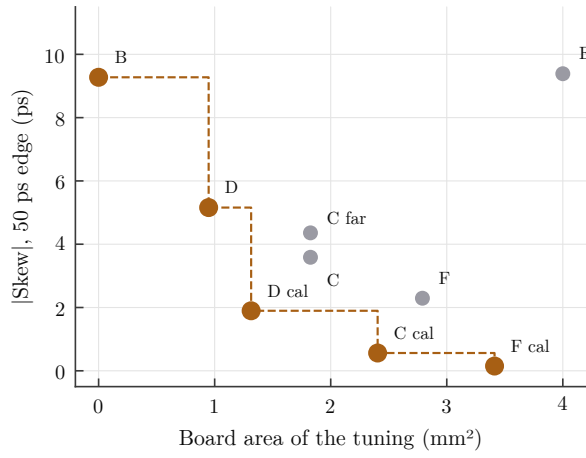


Fig. 21. Two projections of the four-objective comparison. Copper points are non-dominated in that projection, and the dashed line joins them.

solver processes. The README.md lists the commands, and regenerating every figure and table from the stored S-parameters takes a few minutes.

XIII. LIMITATIONS

One width and one pitch, in one microstrip and one stripline stackup. The stripline ports run to the lower plane only, and its plane spacing came from a closed-form estimate. Its Z_{diff} is about 95.6Ω by TDR, so its absolute return loss is worse than microstrip's, which does not affect the skew comparisons. The return plane is a perfect conductor and the copper a conducting sheet, so conductor loss is approximate and roughness is absent. The dielectric loss model has $\tan\delta \propto 1/f$. Lumped ports at the launch are not a connector or a probe. We assume their reflection is common to every variant and cancels in the comparisons. Because the variants' input impedances differ, the assumption may not hold exactly, and it was not tested. It does not hold for absolute return loss. The staircased mesh resolves the smallest features (the $75\mu\text{m}$ chamfers) with under two cells (1.9) at the medium level. The convergence study bounds the effect for B, C, D and the serpentes only. The stripline set was solved on one mesh. Edges faster than 35 ps and frequencies above 30 GHz are outside the simulated band. Common-mode voltage and S_{cd21} are not emissions. Every full-wave result comes from one solver, openEMS. The line model is calibrated on its results, so their agreement is not independent validation. No second solver was used. The mechanisms are not isolated: the corner effect is inferred by elimination, and the serpentine pair varies fold spacing and uncoupled length together.

XIV. CONCLUSIONS

For this fixture at a 20 mil pitch, the U-bend's skew was about 9 ps, as geometry predicts. Matching the lengths did not remove it. A length-matched serpentine removed between 44 and 75 % of the skew in full-wave simulation, and the remainder grew with the number of excursions and the closeness of the folds. A line model with the same lengths missed nearly all of it, and the same copper in stripline behaved the same way.

One field-solver iteration sized the broad and sparse tunings to within 0.15 and 0.56 ps, and the dense accordion needed more. In these simulations more length matching did not close the gap. Size the tuning by what it does electrically, and keep it next to the bend with few excursions set well apart.

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